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Experimental Analysis of Vibration Modes of Plates Using ESPI

Koung-Suk Kim*, Hyun-Chul Jung, Ki-Soo Kang***, Young-June Kang****, Yong-Hoon Cha* and Woon-Gwan Jung*******

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In the world, there are several types of vibration, and these vibrations especially affect the mechanical industry. In this paper, experimental analysis of vibration modes of plates is discussed. Electronic speckle pattern interferometry (ESPI) is one of the optical nondestructive testing techniques. By using the ESPI, vibration modes of plates with various excitation points, ratio of longitudinal and lateral length, and materials are measured and qualitatively compared with the results of theoretical analysis proposed by Warburton. Finally, these vibration modes are quantitatively compared with the result of FEM analysis. The results of this study are as follows: (1) By comparing the theoretical and experimental frequencies, we confirmed qualitatively that deviations of frequencies are within 10% in accuracy. (2) By comparing the experimental vibration mode shapes with the numerical ones of the FEM analysis, we can conclude quantitatively that measuring vibration modes by using the ESPI has high accuracy.

Key Words: Experimental Analysis, Vibration Mode, Rectangular Plate, ESPI, FEM Analysis, Numerical Vibration Mode Shape, Frequency

1. Introduction

Noise and vibration problem during the operation of vehicle engines and machine tools is one of very important issues to machine designers (Suzuki, 1989; Sakada, 1979). To find the practical aspect of these problems, we have measured the vibration modes of rectangular plates clamped at two parallel edges by ESPI (Cloud, 1995; Ennos, 1975; Jones and Wykes, 1989), which is developed in recent years. ESPI (Electronic Speckle Pattern Interferometry) (Kang and Moon, 1996; Kim and Yang, 1994; Kim et al., 1998), developed by adding the television image acquisition and the computer image processing to SPI, is a new method for the analysis of the vibration problems (Kang and Choi, 1996; Rastogi, 1997; Sirohi, 1993).

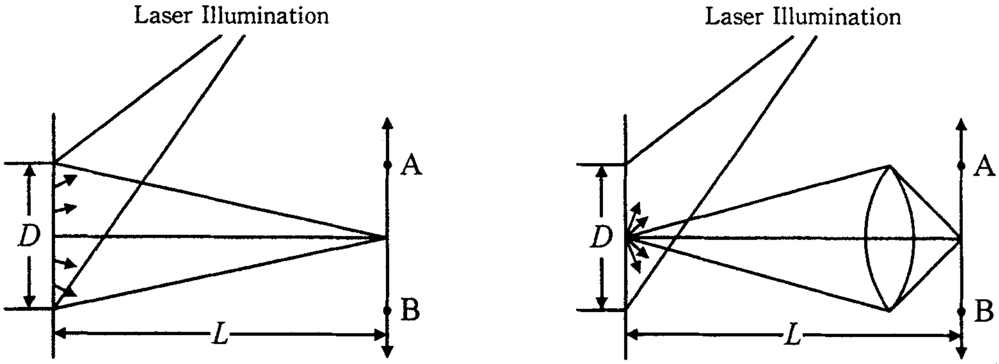
In this paper, vibration modes are measured for each excitation point and aspect ratio of the specimen. Frequencies obtained from the measurement are qualitatively compared with those of the theoretical analysis proposed by Warburton who considered the vibration of rectangular plates with all possible boundary conditions (Warburton, 1954). Shapes of vibration modes are quantitatively compared with those of numerical analysis using FEM.

* Dept. of Mechanical Engineering, Chosun University, Kwangju
** Dept. of Mechanical Engineering, Graduate School, Chosun University, Kwangju
*** Dept. of Mechanical Design Engineering, Graduate School, Chosun University, Kwangju
**** School of Mechanical Engineering, Chonbuk National University, Chonju
***** Dept. of Nuclear Engineering, Chosun University, Kwangju

2. Theory of ESPI

As shown in Fig. 1(a), when the laser light is illuminated to reflective plane, reflective pattern which is known as the 'objective' speckle pattern is formed on the screen.

If the wavelength of the laser light is λ, the diameter of circular region is D and the distance between the object surface and the screen is L, the



(a) (b)

Fig. 1 Formation of (a) objective speckle, (b) subjective speckle.

average size of 'objective' speckle is given by

$$S_{obj} \approx 1.2\lambda L/D. \tag{1}$$

Speckle size is increased when the illuminated area is reduced.

When, alternatively, the laser light illuminated to the objective surface is focused with lens on the screen, speckle patterns are formed. As shown in the Fig. 1(b), this pattern (subjective speckle pattern) is formed when the reflective light is concentrated to the random direction from one point on the illuminated surface and the concentrated lights are interfered by each other. If M is the magnification of the lens and F is the aperture ratio of the lens (F/Number), the size of 'subjective' speckle on the scattering surface is given by

$$S_{subj} \approx 1.2(1+M)\lambda\frac{F}{M}. \tag{2}$$

Speckle phenomenon is occurred by coherency of the laser light. We have tried to eliminate the speckle that is background noise in holography. But, for a few years, we have used that phenomenon in the measurement such as SPI, which gives us easy measurement of the in-plane deformation or vibration and does not require high level of stability of the instrument as compared with HI (holographic interferometry). SPI do not need the recording media with high resolution, whereas HI needs that. While the analysis of the speckle patterns is necessary, very fine fringes formed by the interference of object and holographic reference beam is not needed. The practical speckle size is within $5 \sim 50\ \mu$m. And this size may be changed to fit appropriately to the resolution limitation of the TV system. The major feature of SPI or ESPI is that they can create real-time correlation fringes to be displayed directly on a television monitor without recourse to any form of photographic processing, optical spatial filtering, or plate relocation. Because ESPI records the deformed image with a rate of 30 frames/sec, the effort to prevent the vibration is reduced in some extent. There is no need of the darkroom to process photo. Therefore, we found that ESPI could be easier than HI.

3. Vibration Analysis

Let us consider vibration frequencies of rectangular plates. Warburton (1954) discussed the free transverse vibration of rectangular plates with all possible boundary conditions obtained by combining free, freely-supported, and fixed edges. In this paper, we deal with the case with clamped two parallel edges only.

As shown in Fig. 2, for a rectangular plate with the size of a by b, the vibration form must satisfy the boundary conditions at all edges, also it must satisfy the plate equation as follows;

$$\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2 \partial x^2}+\frac{\partial^4 W}{\partial y^4}+\frac{12\rho(1-\nu^2)}{Eh^2}\frac{\partial^2 W}{\partial t^2}=0, \tag{3}$$

where, W is waveform and ρ, ν, and h are

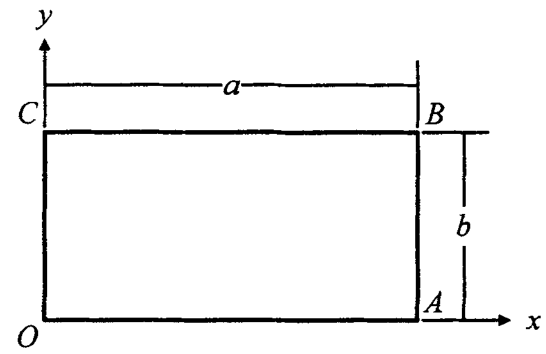


Fig. 2 Details of rectangular plate.

density, Poisson's ratio, and the thickness of the material, respectively. And ω, the displacement at any point (x, y) at time t, is given by

$$w = W \sin \omega t = A\theta(x)\phi(y) \sin \omega t \qquad (4)$$

By the Rayleigh principle, if a suitable waveform W in Eq. 4 is assumed, because it satisfes the boundary conditions approximately, the resulting frequency values that are very near to, but higher than the true values can be obtained. Mode shapes are defined by m and n, number of nodal line on the x and y axes, respectively. Functions for specific beam are substituted into $\theta(x)$ and $\phi(y)$ in Eq. 4. These functions satisfy the boundary conditions of plates with simply supported and fixed edges, while these satisfied approximately those of plates with free edges.

By these functions, α, non-dimensional frequency factor, proportional to frequency is obtained, and α^2 is as follows:

$$\alpha^2 = \frac{\rho a^4 (2\pi f)^2 12(1-\nu^2)}{\pi^4 E h^2}, \qquad (5)$$

where a, h, E, ρ and ν are longitudinal length, thickness, Young's modulus, density and Poisson's ratio of the plate, respectively. When it is found that for all possible boundary conditions, frequencies can be determined from the Eq. 6.

$$\alpha^2 = G_x^4 + G_y^4 \frac{a^4}{b^4} + \frac{2a^2}{b^2}[\nu H_x H_y + (1-\nu) J_x J_y] \qquad (6)$$

where, G_x, G_y, H_x, H_y, J_x and J_y are coefficients dependent on the nodal lines and boundary conditions. Frequency factor α is obtained for any

Table 1 Coefficients in frequency equation.

	Number of m	
	2	3, 4, 5,…
G_x	1.506	$m-1/2$
H_x	1.248	$(m-1/2)^2\left[1-\frac{2}{(m-1/2)\pi}\right]$
J_x	1.248	$(m-1/2)^2\left[1-\frac{2}{(m-1/2)\pi}\right]$

	Number of m			
	0	1	2	3, 4, 5,…
G_y	0	0	1.506	$n-1/2$
H_y	0	0	1.248	$(n-1/2)^2\left[1-\frac{2}{(n-1/2)\pi}\right]$
J_y	0	$\frac{12}{\pi^2}$	5.017	$(n-1/2)^2\left[1-\frac{2}{(n-1/2)\pi}\right]$

aspect ratio, a/b (ratio of longitudinal and lateral length of plate), then frequency is as follows;

$$f = \frac{ah\pi}{a^2}\left[\frac{E}{48\rho(1-\nu^2)}\right]^{1/2} \qquad (7)$$

In the case of the rectangular plate with clamped two parallel edges, each coefficient needed in the frequency equation proposed by Warburton is shown in Table 1.

4. Experimental and Numerical Analysis

In the experimental analysis, the interferometer is operated in time-average mode. In this mode the images are being recorded and added together while the object is vibrating. Vibration fringes can, however, be produced by a time-average subtraction technique, which gives fringes better than those obtained by addition.

The experimental setup used to determine the vibration modes and resonance frequencies of the rectangular plates clamped two parallel edges is shown in Fig. 3. A 8 W Ar^+ laser was used as the light source, and 80 mW power was used in this experiment. The laser beam is divided into two parts by a beam splitter. One beam, called the object beam, travels to the plate and to the CCD

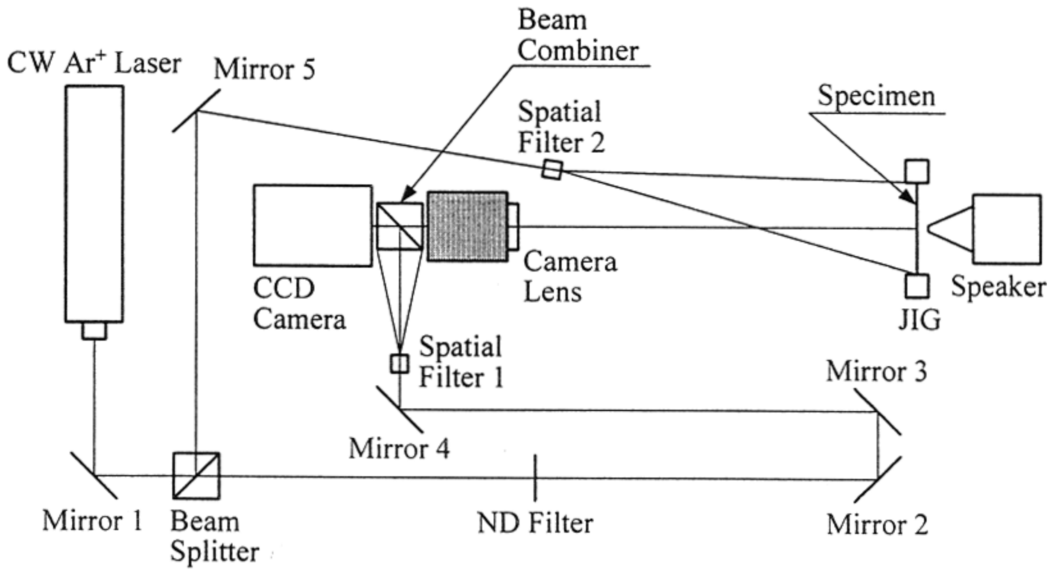


Fig. 3 Schematic of experimental setup.

Table 2 Properties of Specimens.

	E	ν	ρ
STS304	193 GPa	0.3	8.0 g/cm^3
SS400	206 GPa	0.28	7.85 g/cm^3

camera. The other beam, called the reference beam, goes directly to the CCD camera via mirrors. Beam combiner was set in front of the CCD camera. Correlation of two beams is occurred on the CCD array. A video system converts an image formed on the CCD array into an equivalent image on a television monitor screen.

Specimens used were stainless alloy (STS304; in KS) and mild steel (SS400; in KS), and its thickness was 0.8 mm. Aspect ratios of specimens were 2 and 3. Mechanical properties of specimens are as shown in Table 2. The surface of specimens were whitened with paint in order to provide a diffuse optical surface and to obtain fine contrast of the fringe patterns. The dimension and excited points of specimens are shown in Fig. 4.

The plate was excited by means of a 50 W speaker mounted 1 mm behind the specimen. The sinusoidal wave generated by a function generator was provided to the speaker through the amplifier.

The vibration frequencies of the plates were approximately determined by using the time-average ESPI. Correlation fringes in ESPI are observed by the process of video signal subtraction. In the case of the numerical analysis, a general-purpose finite element program, ANSYS, was used to compare with experimental vibration mode shapes.

5. Results and Discussions

By comparing the theoretical natural frequencies obtained by Eq. 7 proposed Warburton with the experimental resonance frequencies, we obtained the deviation between them as follows;

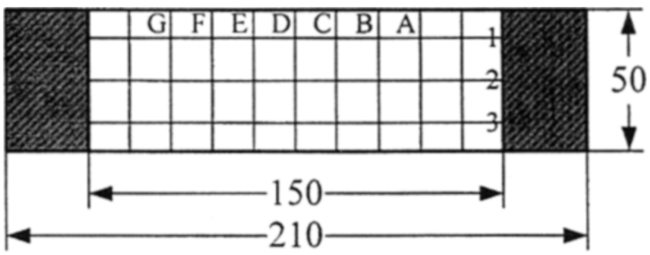
$$\mathrm{Dev}_{T-E}(\%) = \left| \frac{(F_e - F_t)}{F_t} \times 100 \right|, \qquad (8)$$

$$\mathrm{Dev}_{T-F}(\%) = \left| \frac{(F_f - F_t)}{F_t} \times 100 \right|, \qquad (9)$$

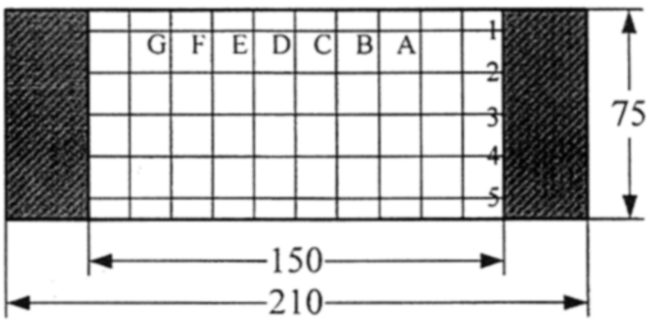
where, Dev_{T-E} and Dev_{T-F} are differences between theoretical and experimental results and between theoretical and numerical analysis results, respectively. F_e, F_t and F_f are results of experimental, theoretical and numerical analyses, respectively.

In the case of STS304 specimens, the mean Dev_{T-E} was about 6% for aspect ratio of 3 and about 7% for aspect ratio of 2. And Dev_{T-F} is less than 3% on each specimen. Dev_{T-E} at lower frequency

(unit: mm)



(a) a/b=3/1



(b) a/b=2/1

Fig. 4 Dimension and excited position of specimens.

(a)

(b)

Fig. 5 Experimental and numerical mode shapes for a STS304 rectangular plate (a/b=3, t=0.8mm, excitation point is (A, 1)) (a) Mode 9 and (b) Mode 19.

was higher than at higher frequency. For aspect ratio of 3, the mean Dev_{T-E} at the horizontally centered excitation points of the specimen was higher than at other points. For aspect ratio of 2, however, the mean Dev_{T-E} has no great difference throughout the excitation points on the specimen. When aspect ratio is 2 or 3, the theoretical, experimental and numerical results at excitation point (A, 1) are shown in Table 3. In Figs. 5 and 6, the mode shapes are shown.

In the case of STS304 specimens, we obtained the similar results. As shown in Eq. 7, frequencies are influenced by Youngs modulus, Poisson's ratio and density. SS400 is milder steel than STS

Table 3 The natural frequencies of rectangular plate when excitation point is (A, 1) (STS304).

STS304 (t=0.8)	a/b=3	Frequencies (Hz)					Differences (%)				
		0	1	2	3	4	0	1	2	3	4
2	F_t	188	409	1873	4822	9291	0	0	0	0	0
	F_e	170	370	1950	4800	9170	9.57	9.54	4.11	0.46	1.30
	F_f	184.6	399.9	1853.6	4793.3	9247	1.81	2.25	1.04	0.56	0.47
3	F_t	519	872	2321	5229	-	0	0	0	0	-
	F_e	460	795	2340	5100	-	11.37	8.83	1.82	2.47	-
	F_f	509.1	865.2	2289.3	5210.4	-	1.91	0.78	1.37	0.36	-
4	F_t	1017	1446	-	-	-	0	0	-	-	-
	F_e	915	1480	-	-	-	10.03	2.35	-	-	-
	F_f	1001.2	1441.5	-	-	-	1.55	0.31	-	-	-
5	F_t	1681	5159	-	6655	-	0	0	-	0	-
	F_e	1670	2140	-	6370	-	0.65	0.88	-	4.28	-
	F_f	1659.8	2153.8	-	6575.3	-	1.26	0.24	-	1.20	-
6	F_t	-	3022	-	-	-	-	0	-	-	-
	F_e	-	2900	-	-	-	-	4.04	-	-	-
	F_f	-	3015.4	-	-	-	-	0.22	-	-	-

STS304 (t=0.8)	a/b=2	Frequencies (Hz)						Differences (%)					
		0	1	2	3	4	5	0	1	2	3	4	5
2	F_t	188	307	933	-	4207	6855	0	0	0	-	0	0
	F_e	172	280	859	-	4120	6600	8.51	8.79	7.93	-	2.07	3.72
	F_f	185.3	302.6	920.4	-	4186.1	6827.7	1.44	1.43	1.35	-	0.50	0.40
3	F_t	519	698	1354	-	-	-	0	0	0	-	-	-
	F_e	450	625	1400	-	-	-	13.29	10.48	3.40	-	-	-
	F_f	511.3	693.3	1337.0	-	-	-	1.48	0.67	1.26	-	-	-
4	F_t	1017	1227	1936	-	-	-	0	0	0	-	-	-
	F_e	905	1260	1910	-	-	-	11.01	2.69	1.34	-	-	-
	F_f	1005.1	1218.8	1917.5	-	-	-	1.17	0.67	0.96	-	-	-
5	F_t	1681	1908	-	-	-	-	0	0	-	-	-	-
	F_e	1650	1900	-	-	-	-	1.84	0.42	-	-	-	-
	F_f	1664.8	1896.6	-	-	-	-	0.96	0.60	-	-	-	-

* - indicates that no experimental results are obtained.

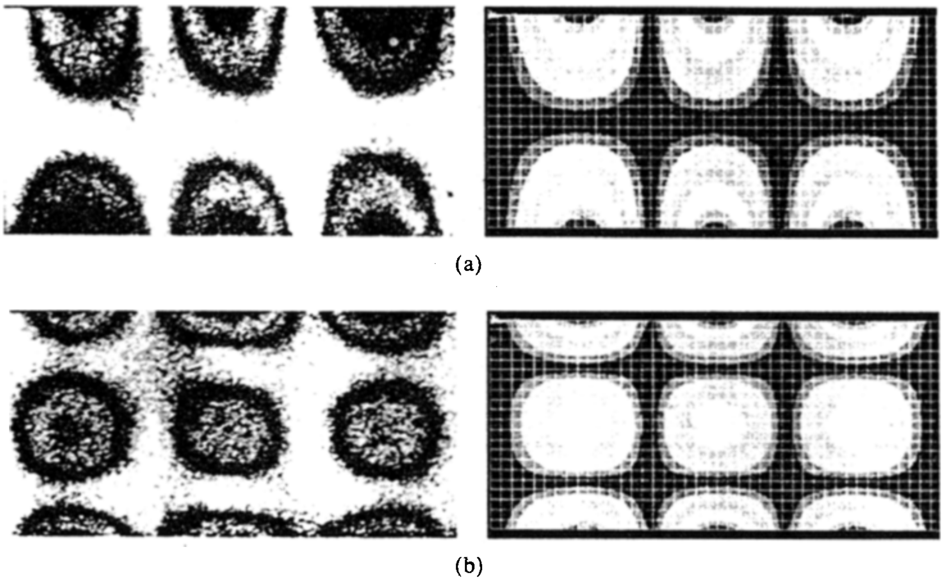


Fig. 6 Experimental and numerical mode shapes for a STS304 rectangular plate (a/b=2, t=0.8mm, excitation point is (A, 1)) (a) Mode 7 and (b) Mode 11.

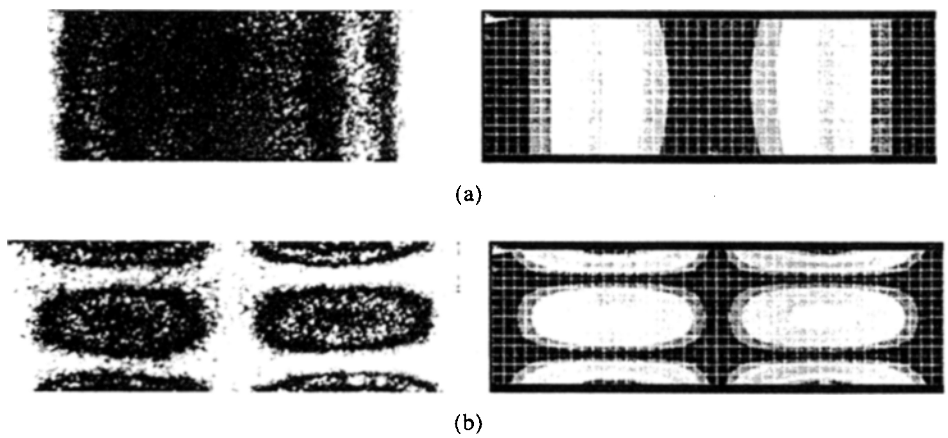


Fig. 7 Experimental and numerical mode shapes for a SS400 rectangular plate (a/b=3, t=1.0mm, excitation point is (A, 1)) (a) Mode 1 and (b) Mode 10.

304. Then, SS400 have higher Youngs modulus than STS304. Therefore, fringe patterns for SS400 specimen was obtained at higher frequencies than the case of STS304 specimen. Mean Dev_{T-E} was about 6% for aspect ratio of 3 and about 9% for aspect ratio of 2. And Dev_{T-F} is less than 3% on the each specimen.

These results are shown that deviation between experimental resonance frequencies and theoretical natural frequencies is greater than those of natural frequencies between numerical and theoretical analyses because air disturbance, noise -vibration of motor pump that used to decrease the temperature of the laser source and the misarrangement of the interferometer affected the experimental analysis.

6. Conclusion

In this paper, the vibration modes measurement of the rectangular plates with clamped two parallel edges by ESPI has been studied and compared

Table 4 The natural frequencies of rectangular plate when excitation point is (A, 1) (SS400).

SS400 (t=1.0)	a/b=3	Frequencies (Hz)				Differences (%)			
		0	1	2	3	0	1	2	3
2	F_t	244	536	2431	6249	0	0	0	0
	F_e	210	490	2500	6050	13.93	8.58	2.84	3.18
	F_f	239.8	523.1	2405.2	6214.8	1.72	2.41	1.06	0.56
3	F_t	672	-	3019	6783	0	-	0	0
	F_e	615	-	3000	6500	8.48	-	0.63	4.17
	F_f	661.0	-	2977.0	6754.2	1.64	-	1.39	0.43
4	F_t	1317	1887	-	7590	0	0	-	0
	F_e	1320	1900	-	7275	0.23	0.69	-	4.15
	F_f	1299.2	1877.0	-	7562.0	1.35	0.53	-	0.37
5	F_t	2178	2813	-	-	0	0	-	-
	F_e	2100	2720	-	-	3.58	3.31	-	-
	F_f	2152.9	2799.4	-	-	1.15	0.48	-	-
6	F_t	3253	-	-	-	0	-	-	-
	F_e	3020	-	-	-	7.16	-	-	-
	F_f	3222.3	-	-	-	0.94	-	-	-

SS400 (t=1.0)	a/b=2	Frequencies (Hz)				Differences (%)			
		0	1	2	3	0	1	2	3
2	F_t	244	401	1212	-	0	0	0	-
	F_e	203	357	1300	-	16.80	10.97	7.26	-
	F_f	240.6	395.4	1195.4	-	1.39	1.40	1.37	-
3	F_t	672	911	1764	-	0	0	0	-
	F_e	495	810	1750	-	26.34	11.09	0.79	-
	F_f	663.8	903.9	1740.0	-	1.22	0.78	1.36	-
4	F_t	1317	1596	2522	-	0	0	0	-
	F_e	1220	1550	2400	-	7.37	2.88	4.84	-
	F_f	1304.3	1586.3	2495.4	-	0.96	0.61	1.06	-
5	F_t	2178	2480	-	-	0	0	-	-
	F_e	1970	2360	-	-	9.55	4.84	-	-
	F_f	2159.8	2465.6	-	-	0.84	0.58	-	-

*- indicates that no experimental results are obtained.

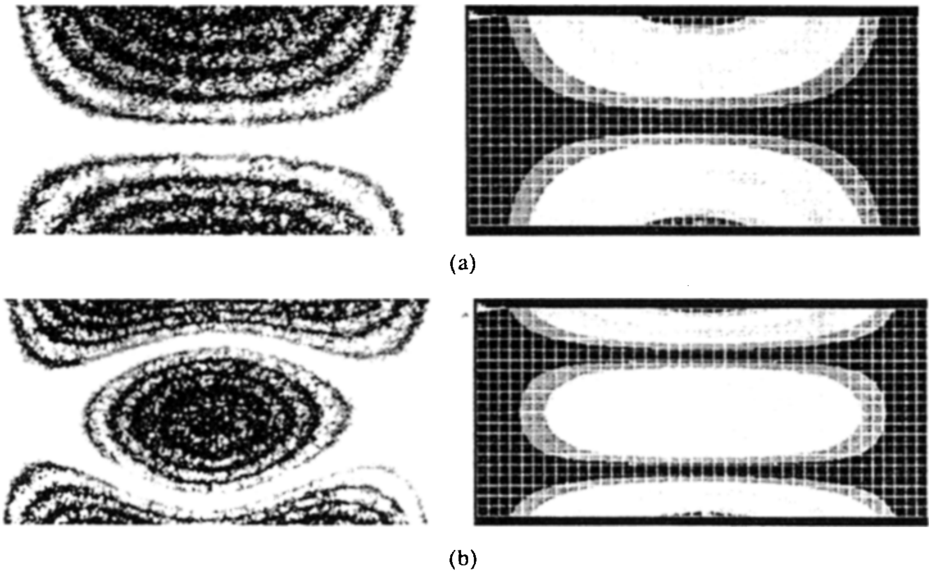


Fig. 8 Experimental and numerical mode shapes for a SS400 rectangular plate (a/b=2, t=1.0mm, excitation point is (A, 1)) (a) Mode 2 and (b) Mode 5.

with theoretical and numerical analyses. It is shown that the experimental results approach the theoretical results within 9%. Mode shapes of the rectangular plates with the aspect ratio of 3 were observed as sensitive as those with the aspect ratio of 2. As SS400 have more flexibility than STS304, the same fringe pattern of vibration modes for STS304 were obtained at lower frequencies. Throughout this study, we found that vibration mode measurement by ESPI could be contributed in the several industrial fields.

Acknowledgement

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